Roll No. ________　　　　　　　　　　　　　　　　**Code No. Q-4313**

SECOND SEMESTER EXAMINATION 2021-22

M.Sc. MATHEMATICS

Paper - III

Topology - II

Time : 3.00 Hrs.　　　　　　　　　　　　　　　　**Max. Marks : 80**

Total No. of Printed Page : 03　　　　　　　　　　　　**Mini. Marks : 29**

Note:- Question paper is divided into three sections. Attempt question of all three section as per direction Distribution of marks is given in each section.

Section 'A'

Very short answer question (in few words)

Q.1 Attempt any six questions from the following : 6x2=12

(i) If X is discrete space where X contains more than one element, prove that X is disconnected.

(ii) Give an example to show that a locally connected space need not be connected.

(iii) show that no infinite discrete space is compact.

(iv) State Lebesque covering lemma.

(v) What do you mean by BWP ?

(vi) Give an example to show that a compact space need not be Hausfroff and also give an to show that a Hausdorffness need not be compact.

(vii) Define free filter and fixed filter.

(viii) Give an example to show that a net can coverage to more than one point.

(ix) Define cluster point of a net.

(x) Define evaluation map.

Section 'B'

Short answer type question (in 200 words)

Q.1 Attempt any four questions from the following : 4x5=20

(i) Prove that a topological space X is connected iff every non-empty proper subset of X has a non-empty frontier.

(ii) Show by means of an exmaple that locally compact space need not be compact.

(iii) Prove that compactness is a topological property.

(iv) Let X be any non-empty set and let f_0 be a non empty subset of X. Then prove that the family $\mathcal{F} = \{F \mid F \supset F_0\}$ is a filter on X.

(v) Consider the topology $\mathcal{J} = \{\varphi, \{a\}, X\}$ for $X = \{a, b, c\}$ and the topology $\mu = \{\phi, \{p\}, \{q\}, \{p,q\}, \{r,s\}, \{p,r,s\}, \{q,r,s,\}, y\}$ for $y = \{p,q,r,s\}$ find a base for the product topology of $X \times Y$.

Section 'C'

Long answer/Essay type question. **4x12=48**

Q.3 Attempt any four questions from the following questions :

(i) (a) Prove that connectedness in a topological property.

(b) Give an example to show that connectedness is not a herediatary property.

(ii) Prove that a topological space X is locally connected iff the components of every open subspace of X are open in X.

(iii) (a) State and prove Heine Borel theorem.

(b) Prove that Cantor's set is compact.

(iv) (a) Prove that a contably compact topological space has BWP.

(b) Prove that every closed subspace of a locally compact space is locally compact.

(v) Prove that a topological space (X, Y) is Hausdorff iff every net in X can coverage to at most one point.

(vi) Prove that every filter on a set X is contained in an ultrafilter on X.

(vii) (a) Prove that the product space $X \times Y$ is connected iff X and Y are connected.

(b) Prove that each projection map $\pi\lambda : X \to X\lambda$ where $X = \underset{\lambda \in a}{X} X\lambda$ is an open map.

(viii) Define Tychonoff cube and Tychonoff space. Prove that every Tychonoff space X can be embedded as a subspace of a cube.

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